

# Form factors and decay rate of $B_c^* \rightarrow D_s l^+ l^-$ decays in the QCD sum rules

K. Zeynali

Department of Sciences, Faculty of Medicine, Ardabil University of Medical Sciences, Ardabil, Iran

( e-mail: k.zeinali@arums.ac.ir)

V. Bashiry

Cyprus International University, Faculty of Engineering, Nicosia, Northern Cyprus, Mersin 10, Turkey

(e-mail: bahiry@ciu.edu.tr)

F. Zolfagharpour

Department of Physics, University of Mohaghegh Ardabili, PO Box 179, Ardabil, Iran

(e-mail: zolfagharpour@uma.ac.ir)

Rare exclusive  $B_c^* \rightarrow D_s l^+ l^-$  decays are analyzed in the framework of the three-point QCD sum rules approach. The two gluon condensate corrections to the correlation function are included and the form factors of this transition are evaluated. Using the form factors, the decay width and integrated decay rate for these decays are also calculated.

## I. INTRODUCTION

The rare flavor-changing neutral-current (FCNC) processes  $\{b \rightarrow s(d)\}$  are widely studied to test the predictions of Standard Model (SM) at loop level and to search for new-physics (NP). The recent theoretical studies can be found in the Refs.[1]-[6].

Various physical observables of leptonic, semileptonic and radiative  $B$  decays have been measured by LHCb. For instance, the form factor, independent observables in the decay  $B^0 \rightarrow K^{*0} \mu^+ \mu^-$  [7] and the  $CP$  asymmetry in  $B^+ \rightarrow K^+ \mu^+ \mu^-$  decays [8] have been measured. More recent measurements in the LHCb for FCNC transitions can be seen in Refs.[9]-[11]. Measurements of various observables at LHCb indicate that SM predictions are in good agreement with the experimental results. Therefore, most of the new physics scenarios are excluded.

Rare  $B_c^* \rightarrow D_s l^+ l^-$  proceeds FCNC transitions. This decay has not yet been measured by LHCb. There is not theoretical studies relevant to the form factors and decay rate of this decay. We try to calculate the form factors and the decay rate of  $B_c^* \rightarrow D_s l^+ l^-$  decay as well. We use the three-point QCD sum rules approach in the calculation of these form factors. The QCD sum rules have widely been used to calculate form factors (some similar studies can be found in Refs.[12]-[19]).

The paper includes 3 sections: In section 2, we recall the effective Hamiltonian and use the three-point QCD sum rules approach to calculate these form factors. In section 3, we will use the numerical values of form factors in order to determine the sensitivity of the decay rate to the invariant dileptonic mass and then present our conclusion.

## II. SUM RULES FOR THE $B_c^* \rightarrow D_s l^+ l^-$ TRANSITION FORM FACTORS

The matrix element of the  $b \rightarrow s \ell^+ \ell^-$  transition can be written as:

$$M(b \rightarrow s \ell^+ \ell^-) = \frac{G_F \alpha}{\sqrt{2} \pi} V_{tb} V_{ts}^* \{c_9^{eff} \mathcal{O}_9 + c_{10} \mathcal{O}_{10} - 2 \frac{m_b}{q^2} c_7^{eff} \mathcal{O}_7\} \quad (1)$$

where

$$\begin{aligned} \mathcal{O}_7 &= \frac{1}{2} [\bar{s} i \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b] [\bar{\ell} \gamma^\mu \ell], \\ \mathcal{O}_9 &= \frac{1}{2} [\bar{s} \gamma_\mu (1 - \gamma_5) b] [\bar{\ell} \gamma^\mu \ell], \\ \mathcal{O}_{10} &= \frac{1}{2} [\bar{s} \gamma_\mu (1 - \gamma_5) b] [\bar{\ell} \gamma^\mu \gamma^5 \ell], \end{aligned}$$

and  $c_7$ ,  $c_9$  and  $c_{10}$  are Wilson coefficients evaluated in the naive dimensional regularization (NDR) scheme at the leading order (LO), next to leading order (NLO) and next-to-next leading order (NNLO) in the SM[20]-[27].  $c_9^{eff}(\hat{s}) = c_9 + Y(\hat{s})$ , where  $Y(\hat{s}) = Y_{\text{pert}}(\hat{s}) + Y_{\text{LD}}$  contains both the perturbative part  $Y_{\text{pert}}(\hat{s})$  and long-distance part  $Y_{\text{LD}}(\hat{s})$ .  $Y(\hat{s})_{\text{pert}}$  in [20] is as follows:

$$Y_{\text{pert}}(\hat{s}) = g(\hat{m}_c, \hat{s}) c_0$$

TABLE I: Masses, total decay widths and branching fractions of dilepton decays of vector charmonium states [28].

$V$	Mass[GeV]	$\Gamma_{\text{tot}}^V[\text{MeV}]$	$\mathcal{B}(V \rightarrow \ell^+ \ell^-)$
$J/\Psi(1S)$	3.097	0.093	$5.9 \times 10^{-2}$ for $\ell = e, \mu$
$\Psi(2S)$	3.686	0.303	$7.82 \times 10^{-3}$ for $\ell = e, \mu$ $3.0 \times 10^{-3}$ for $\ell = \tau$
$\Psi(3770)$	3.773	27.2	$9.6 \times 10^{-6}$ for $\ell = e$
$\Psi(4040)$	4.040	80	$1.1 \times 10^{-5}$ for $\ell = e$
$\Psi(4160)$	4.153	103	$8.1 \times 10^{-6}$ for $\ell = e$
$\Psi(4415)$	4.421	62	$9.4 \times 10^{-6}$ for $\ell = e$

$$\begin{aligned}
& -\frac{1}{2}g(1, \hat{s})(4\bar{c}_3 + 4\bar{c}_4 + 3\bar{c}_5 + \bar{c}_6) - \frac{1}{2}g(0, \hat{s})(\bar{c}_3 + 3\bar{c}_4) \\
& + \frac{2}{9}(3\bar{c}_3 + \bar{c}_4 + 3\bar{c}_5 + \bar{c}_6),
\end{aligned} \tag{2}$$

$$\text{where } c_0 \equiv \bar{c}_1 + 3\bar{c}_2 + 3\bar{c}_3 + \bar{c}_4 + 3\bar{c}_5 + \bar{c}_6, \tag{3}$$

and the function  $g(x, y)$  is defined in [20]. Here,  $\bar{c}_1 - \bar{c}_6$  are the Wilson Coefficients in the leading logarithmic approximation. The relevant Wilson Coefficients are given in [29].  $Y(\hat{s})_{\text{LD}}$  involves  $B_c^* \rightarrow D_s V(\bar{c}c)$  resonances [21, 30, 31], where  $V(\bar{c}c)$  are the vector charmonium states. Following refs. [21, 30],  $Y_{\text{LD}}(\hat{s})$  is as follows:

$$Y_{\text{LD}}(\hat{s}) = -\frac{3\pi}{\alpha_{\text{em}}^2} c_0 \sum_{V=\psi(1s), \dots} \kappa_V \frac{\hat{m}_V \mathcal{B}(V \rightarrow \ell^+ \ell^-) \hat{\Gamma}_{\text{tot}}^V}{\hat{s} - \hat{m}_V^2 + i\hat{m}_V \hat{\Gamma}_{\text{tot}}^V}, \tag{4}$$

where  $\hat{\Gamma}_{\text{tot}}^V \equiv \Gamma_{\text{tot}}^V/m_{B_c^*}$  and  $\kappa_V$  takes different value for different exclusive semileptonic decays. This phenomenological parameters  $\kappa_V$  can be fixed for  $B \rightarrow K^* \ell^+ \ell^-$  decay by equating the naive factorization estimate of the  $B \rightarrow K^* V$  rate and the results of the experimental measurements [29]. For the time being, there is no experimental result on  $B_c^* \rightarrow D_s V(c\bar{c})$ . Thus, we use the results of  $B \rightarrow K^* V$  to estimate the values of  $\kappa_V$  in our numerical calculations i.e.,  $\kappa_V = 1.75$  for  $J/\Psi(1S)$  and  $\kappa_V = 2.43$  for  $\Psi(2S)$ , respectively.

Relevant properties of vector charmonium states are summarized in Table I.

The transition amplitude of the exclusive  $B_c^* \rightarrow D_s \ell^+ \ell^-$  decays is obtained by sandwiching Eq.(1) between the initial meson state  $B_c^*$  and the final meson state  $D_s$  in terms of form factors as follows:

$$\begin{aligned}
M = & \frac{G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{ts}^* \left[ c_9^{\text{eff}} \langle D_s(p_D) | \bar{s} \gamma_\mu (1 - \gamma_5) b | B_c^*(p_B, \varepsilon) \rangle \bar{\ell} \gamma_\mu \ell \right. \\
& + c_{10} \langle D_s(p_D) | \bar{s} \gamma_\mu (1 - \gamma_5) b | B_c^*(p_B, \varepsilon) \rangle \bar{\ell} \gamma_\mu \gamma_5 \ell \\
& \left. - 2c_7^{\text{eff}} \frac{m_b}{q^2} \langle D_s(p_D) | \bar{s} i \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b | B_c^*(p_B, \varepsilon) \rangle \bar{\ell} \gamma_\mu \ell \right],
\end{aligned} \tag{5}$$

where  $\varepsilon$  is the polarization vector of  $B_c^*$  meson,  $p_B$  and  $p_D$  are momentums of the  $B_c^*$  and  $D_s$  mesons, respectively. Taking the Lorentz invariance and parity conservation into account, the matrix elements of the Eq.(5) are parameterized

in terms of the form factors as follows:

$$\begin{aligned} \langle D_s(p_D) | \bar{s}\gamma_\mu(1 - \gamma_5)b | B_c^*(p_B, \varepsilon) \rangle = & A_V(q^2)\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p_B^\alpha p_D^\beta - iA_0(q^2)\varepsilon_\mu^* \\ & - iA_+(q^2)(\varepsilon^* p_D)P_\mu - iA_-(q^2)(\varepsilon^* p_D)q_\mu, \end{aligned} \quad (6)$$

$$\begin{aligned} \langle D_s(p_D) | \bar{s}\sigma_{\mu\nu}q^\nu(1 + \gamma_5)b | B_c^*(p_B, \varepsilon) \rangle = & -T_V(q^2) i\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}p_B^\alpha p_D^\beta \\ & - T_0(q^2)\left\{\varepsilon_\mu^* + \frac{(\varepsilon^* p_D)}{(m_{B_c^*}^2 - m_{D_s}^2)}P_\mu\right\} - T_+(q^2)(\varepsilon^* p_D)\left\{q_\mu - \frac{q^2}{m_{B_c^*}^2 - m_{D_s}^2}P_\mu\right\}, \end{aligned} \quad (7)$$

where  $A_V(q^2)$ ,  $A_0(q^2)$ ,  $A_+(q^2)$ ,  $A_-(q^2)$ ,  $T_V(q^2)$ ,  $T_0(q^2)$  and  $T_+(q^2)$  are the transition form factors.  $P_\mu = (p_B + p_D)_\mu$  and  $q_\mu = (p_B - p_D)_\mu$ , here,  $q$  is transfer momentum or the momentum of the  $Z$  boson (photon).

The transition amplitude in terms of the form factors is as follows:

$$\begin{aligned} M = & \frac{G_F\alpha}{2\sqrt{2}\pi}V_{tb}V_{ts}^*\left\{\bar{\ell}\gamma_\mu\ell\left[A_1\varepsilon_{\mu\nu\alpha\beta}p_B^\alpha p_D^\beta\varepsilon^\nu - A_2(\varepsilon.p_D)P_\mu - A_3(\varepsilon.p_D)q_\mu - A_4\varepsilon_\mu\right]\right. \\ & \left.+ \bar{\ell}\gamma_\mu\gamma_5\ell\left[B_1\varepsilon_{\mu\nu\alpha\beta}p_B^\alpha p_D^\beta\varepsilon^\nu - B_2(\varepsilon.p_D)P_\mu - B_3(\varepsilon.p_D)q_\mu - B_4\varepsilon_\mu\right]\right\}, \end{aligned} \quad (8)$$

where

$$\begin{aligned} A_1 &= -Ic_9A_V - 2C_7T_Vm_b/q^2 \\ A_2 &= Ic_9A_+ + \frac{2c_7m_b}{q^2(m_{B_c^*}^2 - m_{D_s}^2)}(T_0 + q^2T_-) \\ A_3 &= -Ic_9A_- - 2C_7T_-m_b/q^2 \\ A_4 &= Ic_9A_0 - 2C_7T_0m_b/q^2 \\ B_1 &= -IA_Vc_{10}, \quad B_2 = A_+c_{10}, \quad B_3 = A_-c_{10}, \quad B_4 = A_0c_{10} \end{aligned} \quad (9)$$

The decay rate for the  $B_c^* \rightarrow D_s l^+ l^-$  decay is obtained as follows:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2\alpha^2|V_{tb}V_{ts}|^2}{6144\pi^5m_{B_c^*}^3}\lambda^{1/2}(m_{B_c^*}^2, m_{D_s}^2, q^2)v\Delta \quad (10)$$

where

$$v = \sqrt{1 - \frac{4m_\ell^2}{q^2}}, \quad (11)$$

$$\begin{aligned} \Delta = & \lambda(m_{B_c^*}^2, m_{D_s}^2, q^2)\left\{-2|A_1|^2(2m_\ell^2 + q^2) + 2|B_1|^2(4m_\ell^2 - q^2) - \frac{6|B_3|^2m_\ell^2q^2}{m_{B_c^*}^2} + \frac{12Re[B_3B_4^*]m_\ell^2}{m_{B_c^*}^2}\right. \\ & \left.- \frac{12Re[B_2B_3^*]m_\ell^2(m_{B_c^*}^2 - m_{D_s}^2)}{m_{B_c^*}^2}\right\} - \frac{1}{m_{B_c^*}^2q^2}\left\{|A_2|^2\lambda(m_{B_c^*}^2, m_{D_s}^2, q^2)^2(2m_\ell^2 + q^2)m_{B_c^*}^2q^2\right. \\ & \left.+ 2Re[A_2A_4^*](2m_\ell^2 + q^2)\left(m_{B_c^*}^6 - m_{B_c^*}^4(3m_{D_s}^2 + q^2) + m_{B_c^*}^2(3m_{D_s}^4 - 2m_{D_s}^2q^2 - q^4) - (m_{D_s}^2 - q^2)^3\right)\right\} \end{aligned}$$

$$\begin{aligned}
& - |A_4|^2 (2m_\ell^2 + q^2) \left( m_{B_c^*}^4 - 2m_{B_c^*}^2(m_{D_s}^2 - 5q^2) + (m_{D_s}^2 - q^2)^2 \right) - |B_2|^2 \lambda(m_{B_c^*}^2, m_{D_s}^2, q^2) \left( m_{B_c^*}^4 (2m_\ell^2 + q^2) \right. \\
& - 2m_{B_c^*}^2(m_{D_s}^2(2m_\ell^2 + q^2) - 4m_\ell^2 q^2 + q^4) + m_{D_s}^4(2m_\ell^2 + q^2) + m_{D_s}^2(8m_\ell^2 q^2 - 2q^4) - 4m_\ell^2 q^4 + q^6 \Big) \\
& + 2Re[B_2 B_4^*] \lambda(m_{B_c^*}^2, m_{D_s}^2, q^2) \left( m_{B_c^*}^2(2m_\ell^2 + q^2) - m_{D_s}^2(2m_\ell^2 + q^2) - 4m_\ell^2 q^2 + q^4 \right) \\
& \left. - |B_4|^2 \left( m_{B_c^*}^4(2m_\ell^2 + q^2) - 2m_{B_c^*}^2(m_{D_s}^2(2m_\ell^2 + q^2) + 26m_\ell^2 q^2 - 5q^4) + (m_{D_s}^2 - q^2)^2(2m_\ell^2 + q^2) \right) \right\} \quad (12)
\end{aligned}$$

We follow the QCD sum rules approach to calculate the aforementioned form factors. The QCD sum rules start with the following correlation functions:

$$\begin{aligned}
\mathcal{T}_{\mu\nu}^{V-AV}(p_B^2, p_D^2, q^2) &= i^2 \int d^4x d^4y e^{-ip_B x} e^{ip_D y} \langle 0 | T[J_{D_s}(y) J_\mu^{V-AV}(0) J_{\nu B_c^*}(x)] | 0 \rangle, \\
\mathcal{T}_{\mu\nu}^{T-PT}(p_B^2, p_D^2, q^2) &= i^2 \int d^4x d^4y e^{-ip_B x} e^{ip_D y} \langle 0 | T[J_{D_s}(y) J_\mu^{T-PT}(0) J_{\nu B_c^*}(x)] | 0 \rangle, \quad (13)
\end{aligned}$$

where  $J_{D_s}(y) = \bar{c}\gamma_5 s$  and  $J_{\nu B_c^*}(x) = \bar{b}\gamma_\nu c$  are interpolating currents of the  $D_s$  and  $B_c^*$  meson states, respectively.  $J_\mu^{V-AV} = \bar{s}\gamma_\mu(1 - \gamma_5)b$  and  $J_\mu^{T-PT} = \bar{s}\sigma_{\mu\nu}q^\nu(1 + \gamma_5)b$  consist of the vector (V), axial vector (AV), tensor (T) and pseudo tensor (PT) transition currents. The above correlation functions can be re-written by inserting the two complete sets of the  $B_c^*$  and  $D_s$  meson currents with the same quantum numbers into the Eq. (13) as follows:

$$\begin{aligned}
\mathcal{T}_{\mu\nu}^{V-AV}(p_B^2, p_D^2, q^2) &= - \\
& \frac{\langle 0 | J_{D_s} | D_s(p_D) \rangle \langle D_s(p_D) | J_\mu^{V-AV} | B_c^*(p_B, \varepsilon) \rangle \langle B_c^*(p_B, \varepsilon) | J_{\nu B_c^*} | 0 \rangle}{(p_D^2 - m_{D_s}^2)(p_B^2 - m_{B_c^*}^2)} + \dots, \\
\mathcal{T}_{\mu\nu}^{T-PT}(p_B^2, p_D^2, q^2) &= - \\
& \frac{\langle 0 | J_{D_s} | D_s(p_D) \rangle \langle D_s(p_D) | J_\mu^{T-PT} | B_c^*(p_B, \varepsilon) \rangle \langle B_c^*(p_B, \varepsilon) | J_{\nu B_c^*} | 0 \rangle}{(p_D^2 - m_{D_s}^2)(p_B^2 - m_{B_c^*}^2)} + \dots, \quad (14)
\end{aligned}$$

where "... " indicates higher states and continuum contributions. The  $\langle 0 | J_{D_s} | D_s(p_D) \rangle$  and  $\langle B_c^*(p_B, \varepsilon) | J_{\nu B_c^*} | 0 \rangle$  matrix elements are as follows:

$$\langle 0 | J_{D_s} | D_s(p_D) \rangle = -i \frac{f_{D_s} m_{D_s}^2}{m_s + m_c}, \quad \langle B_c^*(p_B, \varepsilon) | J_{\nu B_c^*} | 0 \rangle = f_{B_c^*} m_{B_c^*} \varepsilon_\nu, \quad (15)$$

where  $f_{D_s}$  and  $f_{B_c^*}$  are the leptonic decay constants of the  $D_s$  and  $B_c^*$  mesons, respectively. Using Eq.(6), Eq.(7) and Eq.(15) together with the summation over the polarization of the  $B_c^*$  meson, the Eq.(14) can be re-written as follows:

$$\begin{aligned}
\mathcal{T}_{\mu\nu}^{V-AV}(p_B^2, p_D^2, q^2) &= - \frac{f_{D_s} m_{D_s}^2}{(m_c + m_s)} \frac{f_{B_c^*} m_{B_c^*}}{(p_D^2 - m_{D_s}^2)(p_B^2 - m_{B_c^*}^2)} \times \left[ A_0(q^2) g_{\mu\nu} + A_+(q^2) P_\mu p_{B\nu} \right. \\
& \left. + A_-(q^2) q_\mu p_{B\nu} + i \varepsilon_{\mu\nu\alpha\beta} p_B^\alpha p_D^\beta A_V(q^2) \right] + \text{excited states}, \\
\mathcal{T}_{\mu\nu}^{T-PT}(p_B^2, p_D^2, q^2) &= - \frac{f_{D_s} m_{D_s}^2}{(m_c + m_s)} \frac{f_{B_c^*} m_{B_c^*}}{(p_D^2 - m_{D_s}^2)(p_B^2 - m_{B_c^*}^2)} \times \left[ -i T_0(q^2) g_{\mu\nu} \right. \\
& \left. - i T_+(q^2) q_\mu p_{B\nu} + \varepsilon_{\mu\nu\alpha\beta} p_B^\alpha p_D^\beta T_V(q^2) \right] + \text{excited states}. \quad (16)
\end{aligned}$$

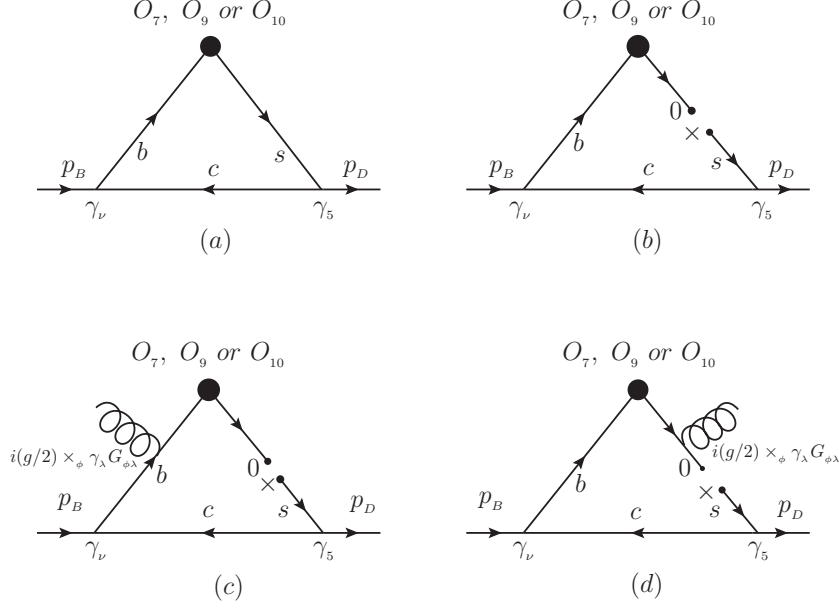


FIG. 1: The bare-loop and light quarks condensates contributions to  $B_c^* \rightarrow D_s l^+ l^-$  transitions

Then, on the QCD or theoretical side, the correlation functions are evaluated in terms of the quarks and gluons parameters by means of the the operator product expansion (OPE). The correlation functions are written as:

$$\begin{aligned}\mathcal{T}_{\mu\nu}^{V-AV}(p_B^2, p_D^2, q^2) &= \mathcal{T}_0^{V-AV} g_{\mu\nu} + \mathcal{T}_+^{V-AV} P_\mu p_{B\nu} + \mathcal{T}_-^{V-AV} q_\mu p_{B\nu} + i\mathcal{T}_V^{V-AV} \varepsilon_{\mu\nu\alpha\beta} p_B^\alpha p_D^\beta, \\ \mathcal{T}_{\mu\nu}^{T-PT}(p_B^2, p_D^2, q^2) &= -i\mathcal{T}_0^{T-PT} g_{\mu\nu} - i\mathcal{T}_+^{T-PT} q_\mu p_{B\nu} + \mathcal{T}_V^{T-PT} \varepsilon_{\mu\nu\alpha\beta} p_B^\alpha p_D^\beta,\end{aligned}\quad (17)$$

where each  $\mathcal{T}_i$  with  $i = 0, +, -$  and  $V$  contains the perturbative and nonpertubative parts as:

$$\mathcal{T}_i = \mathcal{T}_i^{pert} + \mathcal{T}_i^{nonpert}.\quad (18)$$

The perturbative part of the Eq.(18) is the bare-loop diagram given in Fig.1(a). The nonperturbative part consists of the light quark condensates and the two gluon condensates diagrams {see Fig.2(a-f)}. Contributions of the light quark condensates {diagrams shown in Fig.1(b, c, d)} are removed by the application of the double Borel transformations [18]. Therefore, the two gluon condensates diagrams shown in Fig.2(a-f) are described as the first correction.

The bare-loop contributions for each structure in the correlation function are the double dispersion integrals given in the following formula:

$$\mathcal{T}_i^{per} = -\frac{1}{(2\pi)^2} \int du \int ds \frac{\rho_i(s, u, q^2)}{(s - p_B^2)(u - p_D^2)} + \text{subtraction terms}.\quad (19)$$

In the foregoing calculations of the spectral density  $\rho_i(s, u, q^2)$ , Cutkosky Rules are applied to the Feynman Integrals. According to these rules the quark propagators are replaced by Dirac Delta Functions:  $\frac{1}{p^2 - m^2} \rightarrow -2\pi i \delta(p^2 - m^2)$ , which means that all quarks are real.

The integration region in Eq.(19) is restricted by the arguments of the three  $\delta$  functions in  $s, u$  and  $q^2$  space coordinates, where these  $\delta$  functions must be zero at the same time. As a result, the following inequality in  $s, u$  and  $q^2$  space coordinates is obtained:

$$-1 \leq \frac{2su + (s + u - q^2)(m_b^2 - s - m_c^2) + (m_c^2 - m_s^2)2s}{\lambda^{1/2}(m_b^2, s, m_c^2)\lambda^{1/2}(s, u, q^2)} \leq +1 \quad (20)$$

where  $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ac - 2bc - 2ab$ .

Following the required calculations, the spectral densities are obtained as:

$$\begin{aligned} \rho_V^{V-AV} &= N_c I_0(s, u, q^2) \left\{ C_1(m_b - m_c) - (C_2 + 1)m_c + C_2 m_s \right\} \\ \rho_0^{V-AV} &= \frac{N_c}{2} I_0(s, u, q^2) \left\{ -2m_c^3 + 2m_s m_c^2 - [(C_1 + C_2 + 1)(-q^2 + s + u) + 2C_1 s \right. \\ &\quad + 2C_2 u] m_c + m_b[2m_c^2 - 2m_s m_c + 2C_2 u + C_1(-q^2 + s + u)] + m_s[2C_1 s \\ &\quad + C_2(-q^2 + s + u)] \left. \right\} \\ \rho_+^{V-AV} &= \frac{N_c}{2} I_0(s, u, q^2) \left\{ C_1(m_b - 2C_2 m_c - m_c + 2C_2 m_s) \right. \\ &\quad \left. - (2C_2 + 1)(C_2 m_c + m_c - C_2 m_s) \right\} \\ \rho_-^{V-AV} &= \frac{N_c}{2} I_0(s, u, q^2) \left\{ (2C_2 - 1)(C_2 m_c + m_c - C_2 m_s) \right. \\ &\quad \left. + C_1(m_b - 2C_2 m_c - m_c + 2C_2 m_s) \right\} \\ \rho_V^{T-PT} &= 4N_c I_0(s, u, q^2) \left\{ -2sC_1^2 - (m_c^2 - m_s m_c + s + 2C_2(-q^2 + s + u))C_1 \right. \\ &\quad - C_2 m_c^2 + C_2 m_c m_s + m_c m_s + m_b((C_1 + C_2 + 1)m_c - (C_1 + C_2)m_s) - 2C_2^2 u \\ &\quad \left. - C_2 u \right\} \\ \rho_0^{T-PT} &= 2N_c I_0(s, u, q^2) \left\{ C_2 q^2 m_c^2 - C_2 s m_c^2 - 2s m_c^2 + C_2 u m_c^2 + 2u m_c^2 \right. \\ &\quad - C_2 m_s q^2 m_c + m_s q^2 m_c + C_2 m_s s m_c + m_s s m_c - C_2 m_s u m_c - m_s u m_c + C_2 u^2 \\ &\quad - C_2 q^2 u - C_2 s u + C_1[-(q^2 + s - u)m_c^2 + m_s(q^2 + s - u)m_c + s(q^2 - s + u)] \\ &\quad + m_b[m_c((C_1 - C_2 - 1)q^2 + (C_1 + C_2 + 1)(s - u)) + m_s(C_2(q^2 - s + u) \\ &\quad \left. - C_1(q^2 + s - u))] \right\} \\ \rho_+^{T-PT} &= 2N_c I_0(s, u, q^2) \left\{ ((C_1 + C_2 + 2)m_c^2 - (C_1 + C_2 + 1)m_s m_c + 2C_1 C_2 q^2 \right. \end{aligned}$$

$$- m_b((C_1 + C_2 + 1)m_c - (C_1 + C_2)m_s) + C_1s + C_2u) \Big\} \quad (21)$$

where

$$\begin{aligned} I_0(s, u, q^2) &= \frac{1}{4\lambda^{1/2}(s, u, q^2)}, \\ C_1 &= \frac{m_c^2(s - u - q^2) + u(2m_b^2 - s + u - q^2) - m_s^2(s + u - q^2)}{\lambda(s, u, q^2)} \\ C_2 &= \frac{s(2m_s^2 + s - u - q^2) - m_b^2(s + u - q^2) - m_c^2(s - u + q^2)}{\lambda(s, u, q^2)} \\ N_c &= 3. \end{aligned} \quad (22)$$

Now, the aim here is to evaluate the nonperturbative part of the Eq.(18). As has already been mentioned, the contributions of the the light quark condensate diagrams (Fig. 1b,1c and 1d) to the nonperturbative part of the correlation function vanish[18]. Thus, the gluon condensates diagrams shown in Fig.2 are evaluated. The Fock–Schwinger fixed–point gauge [32–34] is used, where  $x^\mu A_\mu^a = 0$ ,  $A_\mu^a$  is the gluon field.

The following integrals must be solved while evaluating the gluon condensate diagrams: [17, 35]:

$$\begin{aligned} I_0[a, b, c] &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2 - m_b^2]^a [(p_B + k)^2 - m_c^2]^b [(p_D + k)^2 - m_s^2]^c}, \\ I_\mu[a, b, c] &= \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu}{[k^2 - m_b^2]^a [(p_B + k)^2 - m_c^2]^b [(p_D + k)^2 - m_s^2]^c}, \\ I_{\mu\nu}[a, b, c] &= \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu}{[k^2 - m_b^2]^a [(p_B + k)^2 - m_c^2]^b [(p_D + k)^2 - m_s^2]^c}, \end{aligned} \quad (23)$$

where  $k$  is the momentum of the spectator quark  $c$ .

The integrals are transferred from Minkowski space–time to Euclidean space–time. Then, the Schwinger representation for the Euclidean propagator is used as in the following:

$$\frac{1}{k^2 + m^2} = \frac{1}{\Gamma(\alpha)} \int_0^\infty d\alpha \alpha^{n-1} e^{-\alpha(k^2 + m^2)}. \quad (24)$$

The Eq.(24) is convenient to perform the Borel transformation, that is:

$$\mathcal{B}_{\hat{p}^2}(M^2) e^{-\alpha p^2} = \delta(1/M^2 - \alpha). \quad (25)$$

Performing integration over loop momentum  $k$  and auxiliary parameters used in the exponential representation of propagators [33], and applying double Borel transformations over  $p_B^2$  and  $p_D^2$ , the transformed form of the integrals (see also [33]) in Eq.(23) can be written as:

$$\hat{I}_0(a, b, c) = i \frac{(-1)^{a+b+c}}{16\pi^2 \Gamma(a) \Gamma(b) \Gamma(c)} (M_1^2)^{2-a-b} (M_2^2)^{2-a-c} \mathcal{U}_0(a+b+c-4, 1-c-b),$$



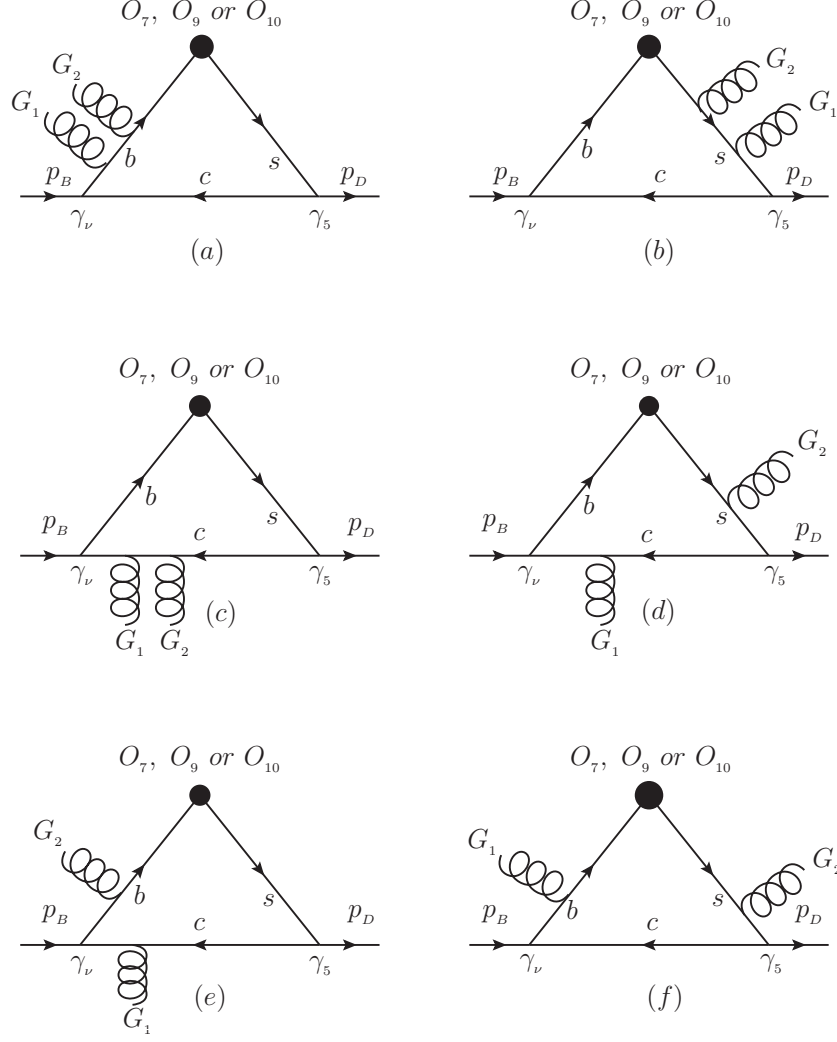


FIG. 2: Gluon condensate contributions to  $B_c^* \rightarrow D_s l^+ l^-$  transitions

$$\hat{I}_\mu(a, b, c) = \hat{I}_1(a, b, c)p_\mu + \hat{I}_2(a, b, c)p'_\mu,$$

$$\begin{aligned} \hat{I}_{\mu\nu}(a, b, c) = & \hat{I}_3(a, b, c)g_{\mu\nu} + \hat{I}_4(a, b, c)p_\mu p_\nu + \hat{I}_5(a, b, c)p'_\mu p'_\nu \\ & + \hat{I}_6(a, b, c)p_\mu p'_\nu + \hat{I}_7(a, b, c)p_\nu p'_\mu, \end{aligned} \quad (26)$$

where

$$\hat{I}_1(a, b, c) = i \frac{(-1)^{a+b+c+1}}{16\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{2-a-b} (M_2^2)^{3-a-c} \mathcal{U}_0(a+b+c-5, 1-c-b),$$

$$\hat{I}_2(a, b, c) = i \frac{(-1)^{a+b+c+1}}{16\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{3-a-b} (M_2^2)^{2-a-c} \mathcal{U}_0(a+b+c-5, 1-c-b),$$

$$\begin{aligned}
\hat{I}_3(a, b, c) &= i \frac{(-1)^{a+b+c+1}}{32\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{3-a-b} (M_2^2)^{3-a-c} \mathcal{U}_0(a+b+c-6, 2-c-b), \\
\hat{I}_4(a, b, c) &= i \frac{(-1)^{a+b+c}}{16\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{2-a-b} (M_2^2)^{4-a-c} \mathcal{U}_0(a+b+c-6, 1-c-b), \\
\hat{I}_5(a, b, c) &= i \frac{(-1)^{a+b+c}}{16\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{4-a-b} (M_2^2)^{2-a-c} \mathcal{U}_0(a+b+c-6, 1-c-b), \\
\hat{I}_6(a, b, c) &= i \frac{(-1)^{a+b+c}}{16\pi^2 \Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{3-a-b} (M_2^2)^{3-a-c} \mathcal{U}_0(a+b+c-6, 1-c-b), \\
\hat{I}_7(a, b, c) &= \hat{I}_6(a, b, c),
\end{aligned} \tag{27}$$

where  $M_1^2$  and  $M_2^2$  are the Borel parameters.

The function  $\mathcal{U}_0(\alpha, \beta)$  is:

$$\mathcal{U}_0(a, b) = \int_0^\infty dy (y + M_1^2 + M_2^2)^a y^b \exp \left[ -\frac{B_{-1}}{y} - B_0 - B_1 y \right],$$

where

$$\begin{aligned}
B_{-1} &= \frac{1}{M_1^2 M_2^2} [m_s^2 M_1^4 + m_b^2 M_2^4 + M_2^2 M_1^2 (m_b^2 + m_s^2 - q^2)], \\
B_0 &= \frac{1}{M_1^2 M_2^2} [(m_s^2 + m_c^2) M_1^2 + M_2^2 (m_b^2 + m_c^2)], \\
B_1 &= \frac{m_c^2}{M_1^2 M_2^2}.
\end{aligned} \tag{28}$$

The Borel transformed form of the phenomenological side {Eq. (16)} and QCD side {Eq. (17)} is calculated. The QCD sum rules for the form factors ( $A_V, A_0, A_+, A_-, T_V, T_0$  and  $T_-$ ) can be achieved by equating the expressions of the phenomenological side {Eq. (16)} and QCD side {Eq. (17)} just after the following Borel transformation:

$$\begin{aligned}
A_i(q^2) &= \frac{(m_s + m_c) e^{m_{B_c^*}^2/M_1^2} e^{m_{D_s}^2/M_2^2}}{f_{B_c^*} m_{B_c^*} f_{D_s} m_{D_s}^2} \left[ \frac{1}{(2\pi)^2} \int_{u_{min}}^{u_0} du \int_{s_{min}}^{s_0} ds \rho_i^{V-AV}(s, u, q^2) e^{-s/M_1^2 - u/M_2^2} \right. \\
&\quad \left. + i \frac{1}{24\pi^2} C^{A_i} < \frac{\alpha_s}{\pi} G^2 > \right],
\end{aligned} \tag{29}$$

$$\begin{aligned}
T_i(q^2) &= \frac{(m_s + m_c) e^{m_{B_c^*}^2/M_1^2} e^{m_{D_s}^2/M_2^2}}{f_{B_c^*} m_{B_c^*}^2 f_{D_s} m_{D_s}^2} \left[ \frac{1}{(2\pi)^2} \int_{u_{min}}^{u_0} du \int_{s_{min}}^{s_0} ds \rho_i^{T-PT}(s, u, q^2) e^{-s/M_1^2 - u/M_2^2} \right. \\
&\quad \left. + i \frac{1}{24\pi^2} C^{T_i} < \frac{\alpha_s}{\pi} G^2 > \right],
\end{aligned} \tag{30}$$

Note that, the Borel transformation suppresses the contributions of higher states and continuum. In addition, the two gluon condensates contributions are  $C^{A_i}$  and  $C^{T_i}$ . The contributions of the aforementioned expressions ( $C^{A_i}$  and  $C^{T_i}$ ) are considered in the numerical analysis. However, since each of these explicit expressions is extremely long, it

is found unnecessary to show them all in this study. Therefore, one of these expressions ( $C^{A_V}$ ) is shown as a sample in Appendix. The  $s_0$  and  $u_0$  are the continuum thresholds in  $s$  and  $u$  channels, respectively. Also  $s_{min} = (m_b + m_c)^2$  and  $u_{min} = (m_s + m_c)^2$ .

### III. NUMERICAL ANALYSIS

From the explicit expressions for the decay rate, it is clear that the main input parameters entering into the expressions are Wilson coefficients  $c_7^{eff}$ ,  $c_9^{eff}$  and  $c_{10}$ , the CKM matrix elements  $|V_{tb}| = 0.77_{-0.24}^{+0.18}$ ,  $|V_{ts}| = (40.6 \pm 2.7) \times 10^{-3}$  [28] and the form factors  $A_V$ ,  $A_0$ ,  $A_+$ ,  $A_-$ ,  $T_V$ ,  $T_0$ ,  $T_-$ . Moreover, the value of the gluon condensate  $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012 \text{ GeV}^4$  [36], the masses and leptonic decay constants,  $m_{B_c^*} = 6.2745 \pm 0.0018 \text{ GeV}$  [28],  $m_{D_s} = 1968.50 \pm 0.32 \text{ MeV}$  [28],  $f_{B_c^*}$  and  $f_{D_s} = (206.7 \pm 8.5 \pm 2.5) \text{ MeV}$  [28], the masses of the quarks  $m_c(\mu = m_c) = 1.275 \pm 0.015 \text{ GeV}$ ,  $m_s(2 \text{ GeV}) \simeq 95 \text{ MeV}$  [28], and  $m_b = (4.18 \pm 0.03) \text{ GeV}$  [28] are necessary for evaluation of the form factors.

In order to reduce the theoretical uncertainties as well as the dependence on the input parameters used in the calculation of the form factors the same set of the input parameters is utilized for calculation of the form factors and the decay constant at the same time. We obtain  $f_{B_c^*} = 428 \pm 35 \text{ MeV}$  by using generic formula given in Ref. [37]. This result is in good agreement with the results of the Ref.[38], which is  $f_{B_c^*} = 415 \pm 31 \text{ MeV}$ . Furthermore, the form factors contain four auxiliary parameters: the Borel mass squares  $M_1^2$  and  $M_2^2$  and the continuum threshold  $s_0$  and  $u_0$ . The physical quantities such as form factors are supposed to be independent or weakly dependent on these auxiliary parameters in the so called "working regions".

The upper bound of the "working region" of  $M_1^2$  and  $M_2^2$  is chosen in a way that the contribution of continuum is less than that of the first resonance. The lower bound of  $M_{1,2}^2$  is fixed so that the contributions proportional to the highest power of  $1/M_{1,2}^2$  are less than about 30%/ of the contributions proportional to the highest power of  $M_{1,2}^2$ . With the aforementioned conditions, we find the stable region for the form factor in the following intervals;  $10 \text{ GeV}^2 \leq M_1^2 \leq 25 \text{ GeV}^2$  and  $4 \text{ GeV}^2 \leq M_2^2 \leq 10 \text{ GeV}^2$ . The continuum thresholds,  $s_0$  and  $u_0$  are determined by the mass of the corresponding ground-state hadron. The value of the  $s_0$  and  $u_0$  must be less than the energy of the first excited states with the same quantum numbers. Hence, the following regions for the  $s_0$  and  $u_0$  are used:  $(m_{B_c^*} + 0.3)^2 \leq s_0 \leq (m_{B_c^*} + 0.7)^2$  and  $(m_{D_s} + 0.3)^2 \leq u_0 \leq (m_{D_s} + 0.7)^2$ .

In order to estimate the decay width of  $B_c^* \rightarrow D_s l^+ l^-$  decays, the  $q^2$  dependency of the form factors  $A_V$ ,  $A_0$ ,  $A_+$ ,  $A_-$ ,  $T_V$ ,  $T_+$  and  $T_0$  in the whole physical region,  $4m_l^2 \leq q^2 \leq (m_{B_c^*} - m_{D_s})^2$ , is required.

The detailed numerical analysis of the form factor depicts that the dependence of the form factors fits into the following function:

$$F(q^2) = \frac{a}{1 - q^2/m_{fit}^2} + \frac{b}{(1 - q^2/m_{fit}^2)^2} \quad (31)$$

The parameters of the fit function are given in Table II:

	$m_{fit}$	$a$	$b$
$A_V(q^2)$	$5.3 \pm 1.2$	$-0.025 \pm 0.005$	$0.046 \pm 0.007$
$A_0(q^2)$	$6.99 \pm 1.5$	$-0.83 \pm 0.13$	$1.25 \pm 0.18$
$A_+(q^2)$	$5.28 \pm 1.2$	$-0.025 \pm 0.005$	$0.047 \pm 0.007$
$A_-(q^2)$	$5.28 \pm 1.2$	$-0.025 \pm 0.005$	$0.047 \pm 0.007$
$T_V(q^2)$	$5.26 \pm 1.2$	$-0.11 \pm 0.017$	$0.21 \pm 0.033$
$T_0(q^2)$	$6.91 \pm 1.14$	$-3.71 \pm 0.48$	$5.62 \pm 1.13$
$T_+(q^2)$	$6.58 \pm 1.08$	$-0.019 \pm 0.003$	$0.029 \pm 0.004$

TABLE II: Parameters appearing in the form factors of the  $B_c^* \rightarrow D_s l^+ l^-$  decay in a four-parameter fit, for  $M_1^2 = 17 \text{ GeV}^2$ ,  $M_2^2 = 17 \text{ GeV}^2$ ,  $s_0 = 46 \text{ GeV}^2$  and  $u_0 = 6 \text{ GeV}^2$

The errors in the numerical calculation shown in Table II stem from the variation of the continuum thresholds, the Borel mass parameter in the given intervals and the uncertainties of the input parameters.

Taking account of the dileptonic invariant mass( $q^2$ ) dependence of the form factors in the kinematical allowed region in the range of  $4m_l^2 \leq q^2 \leq (m_{B_c^*} - m_{D_s})^2$ , we study the the differential decay rate for the  $B_c^* \rightarrow D_s l^+ l^-$  decays. Our results for three different values of the  $q^2$  are presented in Table III. In addition, Fig. (3) depicts the dependence of the differential decay rate on  $q^2$  for full kinematical allowed region.

The branching ratio can be calculated if we know the mean life time of the  $B_c^*$ . There is no experimental data on the mean life time of the  $B_c^*$ . Thus, we ignore about the calculation of the banting ratio. It is worth mentioning that using the theoretical methods like Bethe-Salpeter model [39] and potential model [40], it is possible to calculate the mean life time of the  $B_c^*$  meson and estimate the branching ratio as well.

$q^2(\text{GeV}^2)$	$(d\Gamma/dq^2)(B_c^* \rightarrow D_s \mu^+ \mu^-)$
1	$(3.63 \pm 1.1) \times 10^{-21}$
6	$(4.61 \pm 1.6) \times 10^{-21}$
12	$(8.42 \pm 2.2) \times 10^{-21}$

TABLE III: Values for the decay rate of the  $B_c^* \rightarrow D_s \mu^+ \mu^-$  decay at three different values of the dileptonic invariant mass.

Finally, we calculate the integrated decay rate for the  $B_c^* \rightarrow D_s \mu^+ \mu^-$  decays as follows:

$$\Gamma = \int_{4m_\mu^2}^{(m_{B_c^*} - m_{D_s})^2} \frac{d\Gamma}{dq^2} dq^2 = (3.14 \pm 0.82) \times 10^{-19} \quad (32)$$

To sum up, we investigated the  $B_c^* \rightarrow D_s \ell^+ \ell^-$  decays in the framework of the QCD sum rules approach. The form factors of these decays were obtained in terms of the  $q^2$ . The contributions of the quark condensates in the correlations function found to be zero, so the contributions of the two gluon condensates to the correlations function were evaluated. Finally, we calculated the differential decay width and the integrated decay rate of these decays for the muon channel.

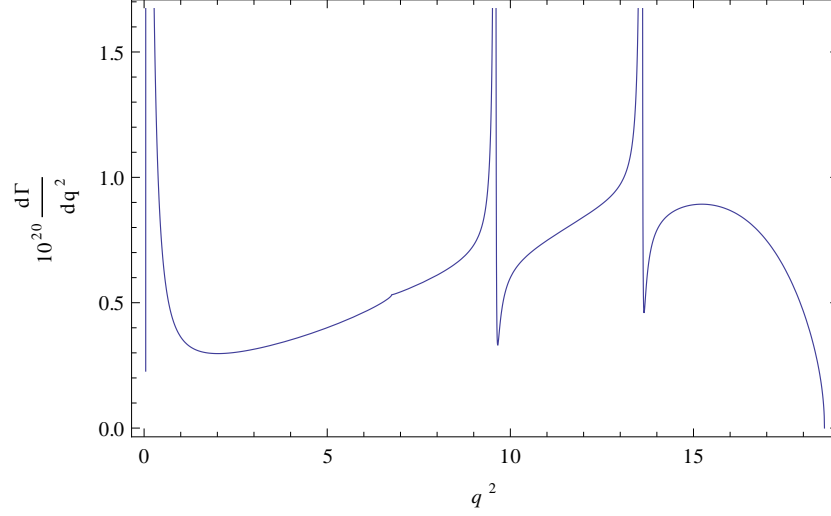


FIG. 3: The dependence of the differential decay rate on  $q^2$  for  $B_c^* \rightarrow D_s \mu^+ \mu^-$  transitions

- 
- [1] D. Cogollo, F. S. Queiroz and P. Vasconcelos, arXiv:1312.0304 [hep-ph].
  - [2] X. -Q. Li, Y. -D. Yang and X. -B. Yuan, arXiv:1311.2786 [hep-ph].
  - [3] A. J. Buras and J. Girrbach, FLAVOUR(267104)-ERC-50 [arXiv:1309.2466 [hep-ph]].
  - [4] A. Ali, A. Y. Parkhomenko and A. V. Rusov, arXiv:1312.2523 [hep-ph].
  - [5] F. o. Richard, arXiv:1312.2467 [hep-ph].
  - [6] G. C. Branco and M. N. Rebelo, PoS Corfu **2012**, 024 (2013) [arXiv:1308.4639 [hep-ph]].
  - [7] RAaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **111**, 191801 (2013) [arXiv:1308.1707 [hep-ex]].
  - [8] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **111**, 151801 (2013) [arXiv:1308.1340 [hep-ex]].
  - [9] RAaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **111**, 101805 (2013) [arXiv:1307.5024 [hep-ex]].
  - [10] M. Vesterinen [LHCb Collaboration], PoS Beauty **2013**, 005 (2013) [arXiv:1306.0092 [hep-ex]].
  - [11] R. Aaij *et al.* [LHCb Collaboration], JHEP **1308**, 131 (2013) [arXiv:1304.6325 [hep-ex]].
  - [12] V. Bashiry, arXiv:1305.6535 [hep-ph].
  - [13] N. Ghahramany and A. R. Houshyar, Acta Phys. Polon. B **44**, no. 9, 1857 (2013).
  - [14] L. -F. Gan, Y. -L. Liu, W. -B. Chen and M. -Q. Huang, Commun. Theor. Phys. **58**, 872 (2012) [arXiv:1212.4671 [hep-ph]].
  - [15] Y. Sarac, K. Azizi and H. Sundu, Nucl. Phys. Proc. Suppl. **245**, 164 (2013).
  - [16] A. Khodjamirian, C. Klein, T. Mannel and N. Offen, Phys. Rev. D **80**, 114005 (2009) [arXiv:0907.2842 [hep-ph]].
  - [17] T. M. Aliev and M. Savci, Eur. Phys. J. C **47**, 413 (2006) [hep-ph/0601267].
  - [18] K. Azizi, F. Falahati, V. Bashiry and S. M. Zebarjad, Phys. Rev. D **77**, 114024 (2008) [arXiv:0806.0583 [hep-ph]].
  - [19] R. S. Marques de Carvalho, F. S. Navarra, M. Nielsen, E. Ferreira and H. G. Dosch, Phys. Rev. D **60**, 034009 (1999) [hep-ph/9903326].
  - [20] A. J. Buras and M. Munz, Phys. Rev. D **52**, 186 (1995) [arXiv:hep-ph/9501281].
  - [21] C. S. Lim, T. Morozumi and A. I. Sanda, Phys. Lett. B **218**, 343 (1989).
  - [22] W. S. Hou, R. S. Willey and A. Soni, Phys. Rev. Lett. **58** (1987) 1608; *ibid* **60** (1988) 2337 *Erratum*.
  - [23] N. G. Deshpande and J. Trampetic, Phys. Rev. Lett. **60** (1988) 2583.
  - [24] M. Jezabek and J. H. Kühn, Nucl. Phys. **B320** (1989) 20.
  - [25] M. Misiak, Nucl. Phys. **B393** 1993 23.
  - [26] M. Misiak, Nucl. Phys. **B439** 461(E) 1995.
  - [27] T. Huber, E. Lunghi, M. Misiak and D. Wyler, Nucl. Phys. **B740** (2006) 105.
  - [28] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).
  - [29] A. Ali, P. Ball, L. T. Handoko and G. Hiller, Phys. Rev. D **61**, 074024 (2000) [arXiv:hep-ph/9910221].
  - [30] A. Ali, T. Mannel and T. Morozumi, Phys. Lett. B **273**, 505 (1991).
  - [31] F. Kruger and L. M. Sehgal, Phys. Lett. B **380**, 199 (1996) [arXiv:hep-ph/9603237].
  - [32] V. A. Fock, Sov. Phys. **12**, 404 (1937).
  - [33] J. Schwinger, Phys. Rev. **82**, 664 (1951).
  - [34] V. Smilga, Sov. J. Nucl. Phys. **35**, 215 (1982).

- [35] V. V. Kiselev, A. K. Likhoded, A. I. Onishchenko, Nucl. Phys. **B 569** (2000) 473.
- [36] M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, Nucl. Phys. **B147** (1979) 385.
- [37] V. Bashiry, K. Azizi and S. Sultansoy, Phys. Rev. D **84**, 036006 (2011) [arXiv:1104.2879 [hep-ph]].
- [38] Z. -G. Wang, Eur. Phys. J. A **49** (2013) 131 [arXiv:1203.6252 [hep-ph]].
- [39] A. Abd El-Hady, M. A. K. Lodhi and J. P. Vary, Phys. Rev. D **59**, 094001 (1999) [hep-ph/9807225].
- [40] V. V. Kiselev, A. E. Kovalsky and A. I. Onishchenko, Phys. Rev. D **64**, 054009 (2001) [hep-ph/0005020].

### Appendix

In this section, we present the explicit expressions for the coefficients  $C^{Av}$  corresponding to the gluon condensates contributions of  $g_{\mu\nu}$  structure entering to the expressions for the form factors in Eq.(29).

$$\begin{aligned}
C^{Av} = & (8m_b - 16m_s + 16m_c)I(1, 1, 2) + (24m_s^3 - 40m_b m_s^2 + 16m_c m_s^2 \\
& + 24m_b^2 m_s + 32m_c^2 m_s - 32q^2 m_s - 16m_b m_c m_s - 32m_c^3 - 16m_b m_c^2 - 8m_c q^2)I(1, 1, 3) \\
& + (-48m_c^2 m_s^3 + 48m_c^3 m_s^2 + 48m_b m_c^2 m_s^2 - 48m_b m_c^3 m_s + 24m_c^2 q^2 m_s)I(1, 1, 4) \\
& - 56m_s I(1, 2, 1) + (8m_s^3 - 8m_b m_s^2 + 8m_c^2 m_s - 32m_b m_c m_s - 16m_c^3 - 8m_b m_c^2 \\
& - 8m_c q^2)I(1, 2, 2) + (24m_s^5 - 24m_b m_s^4 - 40m_c m_s^4 + 8m_c^2 m_s^3 - 16q^2 m_s^3 + 32m_b m_c m_s^3 \\
& - 8m_b m_c^2 m_s^2 - 8m_c q^2 m_s^2 + 16m_c^4 m_s + 16m_b m_c^3 m_s - 8m_c^2 q^2 m_s - 8m_c^5 - 16m_b m_c^4 \\
& - 8m_c^3 q^2)I(1, 2, 3) + (-64m_s^3 + 16m_b m_s^2 + 16m_c m_s^2 + 24m_b^2 m_s + 24m_c^2 m_s + 8q^2 m_s \\
& - 16m_b m_c m_s - 24m_b m_c^2 - 24m_b^2 m_c)I(1, 3, 1) + (16m_s^5 - 16m_b m_s^4 - 16m_c m_s^4 \\
& + 8m_c^2 m_s^3 - 24q^2 m_s^3 + 16m_b m_c m_s^3 - 32m_c^3 m_s^2 - 8m_b m_c^2 m_s^2 + 24m_c^4 m_s + 32m_b m_c^3 m_s \\
& - 16m_c^2 q^2 m_s - 24m_b m_c^4)I(1, 3, 2) + (8m_s^7 - 8m_b m_s^6 - 16m_c m_s^6 - 8m_c^2 m_s^5 - 8q^2 m_s^5 \\
& + 16m_b m_c m_s^5 + 32m_c^3 m_s^4 + 8m_b m_c^2 m_s^4 - 8m_c^4 m_s^3 - 32m_b m_c^3 m_s^3 + 16m_c^2 q^2 m_s^3 - 16m_c^5 m_s^2 \\
& + 8m_b m_c^4 m_s^2 + 8m_c^6 m_s + 16m_b m_c^5 m_s - 8m_c^4 q^2 m_s - 8m_b m_c^6)I(1, 3, 3) + (-48m_s^5 \\
& + 48m_b m_s^4 + 48m_c m_s^4 + 24q^2 m_s^3 - 48m_b m_c m_s^3)I(1, 4, 1) + (8m_c - 8m_s)I(2, 1, 1) \\
& + (24m_s m_b^2 + 16m_s m_c m_b - 8m_s m_c^2 - 16m_s q^2)I(2, 1, 2) + (24m_s m_b^4 - 24m_s m_c^2 m_b^2 \\
& - 16m_s^2 m_c m_b^2 + 48m_s m_c^3 m_b - 48m_s^2 m_c^2 m_b + 32m_s^2 q^2 m_b - 32m_s m_c q^2 m_b + 48m_s m_c^4 \\
& + 24m_s q^4 - 32m_s^2 m_c^3 - 88m_s m_c^2 q^2 + 16m_s^2 m_c q^2)I(2, 1, 3) + (-16m_b^3 + 16m_s m_b^2 \\
& - 8m_c m_b^2 - 16m_s^2 m_b - 8q^2 m_b + 16m_s m_c m_b + 16m_s^3 - 8m_s q^2 - 8m_s^2 m_c)I(2, 2, 1) \\
& + (16m_s^5 - 16m_b m_s^4 - 16m_c m_s^4 + 8m_b^2 m_s^3 - 24q^2 m_s^3 + 16m_b m_c m_s^3 - 32m_b^3 m_s^2 \\
& - 8m_b^2 m_c m_s^2 + 24m_b^4 m_s - 16m_b^2 q^2 m_s + 32m_b^3 m_c m_s - 24m_b^4 m_c)I(2, 3, 1) \\
& + (-32m_b^3 + 32m_s m_b^2 - 16m_c m_b^2 + 16m_s^2 m_b - 8q^2 m_b - 16m_s m_c m_b - 72m_s^3 \\
& + 24m_s m_c^2 + 16m_s q^2 + 56m_s^2 m_c)I(3, 1, 1) + (16m_s m_b^4 - 32m_s^2 m_b^3 + 16m_s m_c m_b^3 \\
& + 8m_s m_c^2 m_b^2 - 40m_s q^2 m_b^2 - 16m_s^2 m_c m_b^2 + 32m_s m_c^3 m_b - 16m_s^2 m_c^2 m_b + 16m_s^2 q^2 m_b \\
& - 32m_s m_c q^2 m_b + 24m_s m_c^4 + 24m_s q^4 - 32m_s^2 m_c^3 - 48m_s m_c^2 q^2 + 32m_s^2 m_c q^2)I(3, 1, 2) \\
& + (8m_s m_b^6 - 16m_s^2 m_b^5 + 16m_s m_c m_b^5 - 8m_s m_c^2 m_b^4 - 24m_s q^2 m_b^4 - 16m_s^2 m_c m_b^4 \\
& - 32m_s m_c^3 m_b^3 + 32m_s^2 m_c^2 m_b^3 + 32m_s^2 q^2 m_b^3 - 32m_s m_c q^2 m_b^3 - 8m_s m_c^4 m_b^2 + 24m_s q^4 m_b^2 \\
& + 32m_s^2 m_c^3 m_b^2 - 16m_s m_c^2 q^2 m_b^2 + 32m_s^2 m_c q^2 m_b^2 + 16m_s m_c^5 m_b - 16m_s^2 m_c^4 m_b
\end{aligned} \tag{33}$$

$$\begin{aligned}
& - 16m_s^2q^4m_b + 16m_sm_cq^4m_b - 32m_sm_c^3q^2m_b + 32m_s^2m_c^2q^2m_b + 8m_sm_c^6 - 8m_sq^6 \\
& - 16m_s^2m_c^5 + 24m_sm_c^2q^4 - 16m_s^2m_cq^4 - 24m_sm_c^4q^2 + 32m_s^2m_c^3q^2)I(3, 1, 3) \\
& + (-8m_b^5 + 16m_sm_b^4 - 16m_cm_b^4 - 8q^2m_b^3 + 16m_sm_cm_b^3 + 8m_s^3m_b^2 - 8m_sq^2m_b^2 \\
& - 8m_s^2m_cm_b^2 - 40m_b^4m_b - 8m_s^2q^2m_b + 32m_s^3m_cm_b + 24m_s^5 - 16m_s^3q^2 \\
& - 24m_b^4m_c)I(3, 2, 1) + (8m_s^7 - 16m_bm_s^6 - 8m_cm_s^6 - 8m_b^2m_s^5 - 8q^2m_s^5 + 16m_bm_cm_s^5 \\
& + 32m_b^3m_s^4 + 8m_b^2m_cm_s^4 - 8m_b^4m_s^3 + 16m_b^2q^2m_s^3 - 32m_b^3m_cm_s^3 - 16m_b^5m_s^2 \\
& + 8m_b^4m_cm_s^2 + 8m_b^6m_s - 8m_b^4q^2m_s + 16m_b^5m_cm_s - 8m_b^6m_c)I(3, 3, 1) + (-144m_s^2m_b^3 \\
& + 144m_sm_cm_b^3 + 144m_s^3m_b^2 - 72m_sq^2m_b^2 - 144m_s^2m_cm_b^2)I(4, 1, 1) \\
& + (8m_bq^2 - 8m_cq^2)I_1(1, 1, 3) + (24m_sm_c^2q^2 - 24m_bm_c^2q^2)I_1(1, 1, 4) \\
& + (8m_bq^2 + 24m_sq^2)I_1(1, 3, 1) + (24m_s^3q^2 - 24m_bm_s^2q^2)I_1(1, 4, 1) \\
& + (8m_bq^2 - 8m_cq^2)I_1(2, 1, 2) + (-16m_bq^4 + 8m_cq^4 - 24m_c^3q^2 + 24m_bm_c^2q^2 \\
& + 8m_b^2m_cm_c^2)I_1(2, 1, 3) + (16q^2m_b^3 - 8m_sq^2m_b^2 + 8m_s^2q^2m_b - 16m_s^3q^2)I_1(2, 3, 1) \\
& + (40m_sq^2 - 16m_bq^2)I_1(3, 1, 1) + (-16m_bq^4 + 24m_b^3q^2 + 16m_bm_c^2q^2)I_1(3, 1, 2) \\
& + (8m_bq^6 - 16m_b^3q^4 - 16m_bm_c^2q^4 + 8m_b^5q^2 + 8m_bm_c^4q^2 - 16m_b^3m_c^2q^2)I_1(3, 1, 3) \\
& + (16q^2m_b^3 - 8m_sq^2m_b^2 + 8m_s^2q^2m_b - 16m_s^3q^2)I_1(3, 2, 1) + (8q^2m_b^5 - 8m_sq^2m_b^4 \\
& - 16m_s^2q^2m_b^3 + 16m_s^3q^2m_b^2 + 8m_s^4q^2m_b - 8m_s^5q^2)I_1(3, 3, 1) \\
& + (72m_b^3q^2 - 72m_b^2m_sq^2)I_1(4, 1, 1) + (-8m_sq^2 - 16m_cq^2)I_2(1, 1, 3) \\
& + (24m_sm_c^2q^2 - 24m_c^3q^2)I_2(1, 1, 4) + (8m_sq^2 - 8m_cq^2)I_2(1, 2, 2) + (-16q^2m_s^3 \\
& + 8m_cq^2m_s^2 - 8m_c^2q^2m_s + 16m_c^3q^2)I_2(1, 2, 3) + (24m_sq^2 + 8m_cq^2)I_2(1, 3, 1) \\
& + (-16q^2m_s^3 + 8m_cq^2m_s^2 - 8m_c^2q^2m_s + 16m_c^3q^2)I_2(1, 3, 2) + (-8q^2m_s^5 + 8m_cq^2m_s^4 \\
& + 16m_c^2q^2m_s^3 - 16m_c^3q^2m_s^2 - 8m_c^4q^2m_s + 8m_c^5q^2)I_2(1, 3, 3) \\
& + (24m_s^3q^2 - 24m_s^2m_cq^2)I_2(1, 4, 1) + (8m_cq^2 - 8m_bq^2)I_2(2, 1, 2) \\
& + (40m_c^3q^2 - 16m_cq^4)I_2(2, 1, 3) + (-8m_bq^2 + 48m_sq^2 - 40m_cq^2)I_2(3, 1, 1) \\
& + (8m_bq^4 - 16m_cq^4 - 8m_b^3q^2 + 16m_c^3q^2 - 8m_bm_c^2q^2 + 8m_b^2m_cm_c^2)I_2(3, 1, 2) \\
& + (8m_cq^6 - 16m_c^3q^4 - 16m_b^2m_cq^4 + 8m_c^5q^2 - 16m_b^2m_c^3q^2 + 8m_b^4m_cm_c^2)I_2(3, 1, 3) \\
& + (72m_b^2m_cq^2 - 72m_b^2m_sq^2)I_2(4, 1, 1) \\
& + D_3^0 \left\{ 8m_sI(3, 3, 1) + (8m_c - 8m_b)I_1(3, 3, 1) \right\} \\
& + D_0^3 \left\{ 8m_bI(1, 3, 3) + (8m_c - 8m_s)I_2(1, 3, 3) \right\}
\end{aligned}$$



$$\begin{aligned}
& + D_0^2 \left\{ (-24m_b + 8m_c - 16m_s)I(1, 2, 3) + (8m_c - 24m_b)I(1, 3, 2) \right. \\
& + (8m_c^3 - 24m_b m_c^2 - 16m_s m_c^2 + 8m_s^2 m_c - 8q^2 m_c + 16m_b m_s m_c - 24m_b m_s^2)I(1, 3, 3) \\
& + (8m_s - 16m_c)I_2(1, 2, 3) + (16m_s - 8m_c)I_2(1, 3, 2) \\
& \left. + (-16m_c^3 + 16m_s m_c^2 - 16m_s^2 m_c - 8q^2 m_c + 16m_s^3 + 8m_s q^2)I_2(1, 3, 3) \right\} \\
& + D_2^0 \left\{ D_0^1 \left[ 8m_c I(3, 3, 1) + (8m_c - 8m_b)I_1(3, 3, 1) + (16m_c - 16m_b)I_2(3, 3, 1) \right] \right. \\
& + (8m_c - 24m_s)I(2, 3, 1) + (-16m_b + 8m_c - 24m_s)I(3, 2, 1) + (8m_c^3 - 16m_b m_c^2 \\
& - 24m_s m_c^2 + 8m_b^2 m_c - 8q^2 m_c + 16m_b m_s m_c - 24m_b^2 m_s)I(3, 3, 1) \\
& + (16m_b - 8m_c)I_1(2, 3, 1) + (8m_b - 16m_c)I_1(3, 2, 1) \\
& \left. + (16m_b^3 - 16m_c m_b^2 + 16m_c^2 m_b + 8q^2 m_b - 16m_c^3 - 8m_c q^2)I_1(3, 3, 1) \right\} \\
& + D_0^1 \left\{ D_0^2 \left[ 8m_c I(1, 3, 3) + (16m_c - 16m_s)I_1(1, 3, 3) + (8m_c - 8m_s)I_2(1, 3, 3) \right] \right. \\
& + D_0^1 \left[ (-16m_c - 8m_s)I(1, 2, 3) - 16m_c I(1, 3, 2) + (-16m_c^3 - 16m_s^2 m_c)I(1, 3, 3) \right. \\
& - 16m_c I(2, 3, 1) + (-8m_b - 16m_c)I(3, 2, 1) + (-16m_c^3 - 16m_b^2 m_c)I(3, 3, 1) \\
& + (16m_s - 32m_c)I_1(1, 2, 3) + (32m_s - 16m_c)I_1(1, 3, 2) + (-32m_c^3 + 32m_s m_c^2 \\
& - 32m_s^2 m_c + 32m_s^3)I_1(1, 3, 3) + (16m_b - 8m_c)I_1(2, 3, 1) + (8m_b - 16m_c)I_1(3, 2, 1) \\
& + (16m_b^3 - 16m_c m_b^2 + 16m_c^2 m_b - 16m_c^3)I_1(3, 3, 1) + (8m_s - 16m_c)I_2(1, 2, 3) \\
& + (16m_s - 8m_c)I_2(1, 3, 2) + (-16m_c^3 + 16m_s m_c^2 - 16m_s^2 m_c + 16m_s^3)I_2(1, 3, 3) \\
& + (32m_b - 16m_c)I_2(2, 3, 1) + (16m_b - 32m_c)I_2(3, 2, 1) + (32m_b^3 - 32m_c m_b^2 \\
& + 32m_c^2 m_b - 32m_c^3)I_2(3, 3, 1) \left. \right] + (8m_c + 8m_s)I(1, 1, 3) - 24m_c m_s^2 I(1, 1, 4) \\
& + 8m_s I(1, 2, 2) + (16m_c^3 + 8m_s m_c^2 + 8m_s^2 m_c + 8m_s^3)I(1, 2, 3) \\
& + (24m_s - 32m_c)I(1, 3, 1) + (24m_c^3 + 16m_s^2 m_c)I(1, 3, 2) + (8m_c^5 - 16m_s^2 m_c^3 \\
& + 8m_s^4 m_c)I(1, 3, 3) - 24m_c^3 I(1, 4, 1) + (24m_b - 16m_c + 16m_s)I(2, 2, 1) + (-16m_c^3 \\
& + 32m_b m_c^2 + 40m_s m_c^2 - 32m_b^2 m_c + 16q^2 m_c - 32m_b m_s m_c + 48m_b^2 m_s)I(2, 3, 1) \\
& + (24m_b - 64m_c + 24m_s)I(3, 1, 1) + (24m_b^3 - 32m_c m_b^2 + 40m_s m_b^2 + 56m_c^2 m_b \\
& + 8q^2 m_b - 32m_c m_s m_b - 32m_c^3 + 16m_c q^2 + 48m_c^2 m_s)I(3, 2, 1) + (-16m_c^5 \\
& + 32m_b m_c^4 + 24m_s m_c^4 - 32m_b^2 m_c^3 + 16q^2 m_c^3 - 32m_b m_s m_c^3 + 32m_b^3 m_c^2 + 16m_b^2 m_s m_c^2 \\
& - 16m_b^4 m_c + 16m_b^2 q^2 m_c - 32m_b^3 m_s m_c + 24m_b^4 m_s)I(3, 3, 1) + 72m_b^2 m_c I(4, 1, 1) \\
& + (-8m_b + 16m_c + 40m_s)I_1(1, 1, 3) + (48m_s^3 + 24m_b m_s^2 - 72m_c m_s^2)I_1(1, 1, 4)
\end{aligned}$$

$$\begin{aligned}
& + (16m_s - 16m_c)I_1(1, 2, 2) + (32m_c^3 - 16m_sm_c^2 + 16m_s^2m_c - 32m_s^3)I_1(1, 2, 3) \\
& + (-8m_b - 72m_c - 16m_s)I_1(1, 3, 1) + (32m_c^3 - 16m_sm_c^2 + 16m_s^2m_c \\
& - 32m_s^3)I_1(1, 3, 2) + (16m_c^5 - 16m_sm_c^4 - 32m_s^2m_c^3 + 32m_s^3m_c^2 + 16m_s^4m_c \\
& - 16m_s^5)I_1(1, 3, 3) + (-72m_c^3 + 24m_bm_c^2 + 48m_sm_c^2)I_1(1, 4, 1) \\
& + (8m_b - 8m_s)I_1(2, 1, 2) + (-56m_s^3 - 24m_bm_s^2 - 8m_b^2m_s + 24q^2m_s \\
& + 16m_bq^2)I_1(2, 1, 3) + (-16m_b^3 + 8m_cm_b^2 - 8m_c^2m_b - 16q^2m_b + 16m_c^3 \\
& + 8m_cq^2)I_1(2, 3, 1) + (32m_b - 136m_c + 80m_s)I_1(3, 1, 1) + (-8m_b^3 - 16m_sm_b^2 \\
& - 32m_s^3 + 32m_sq^2)I_1(3, 1, 2) + (-8m_b^5 - 16m_sm_b^4 + 16m_s^2m_b^3 + 16q^2m_b^3 + 32m_s^3m_b^2 \\
& + 32m_sq^2m_b^2 - 8m_s^4m_b - 8q^4m_b + 16m_s^2q^2m_b - 16m_s^5 - 16m_sq^4 + 32m_s^3q^2)I_1(3, 1, 3) \\
& + (-16m_b^3 + 8m_cm_b^2 - 8m_c^2m_b - 8q^2m_b + 16m_c^3 + 16m_cq^2)I_1(3, 2, 1) + (-8m_b^5 \\
& + 8m_cm_b^4 + 16m_c^2m_b^3 - 16q^2m_b^3 - 16m_c^3m_b^2 + 16m_cq^2m_b^2 - 8m_c^4m_b - 16m_c^2q^2m_b \\
& + 8m_c^5 + 16m_c^3q^2)I_1(3, 3, 1) + (-72m_b^3 + 216m_cm_b^2 - 144m_sm_b^2)I_1(4, 1, 1) \\
& + (8m_c + 16m_s)I_2(1, 1, 3) + (24m_s^3 - 24m_cm_s^2)I_2(1, 1, 4) + (8m_s - 8m_c)I_2(1, 2, 2) \\
& + (16m_c^3 - 8m_sm_c^2 + 8m_s^2m_c - 16m_s^3)I_2(1, 2, 3) + (-24m_c - 8m_s)I_2(1, 3, 1) \\
& + (16m_c^3 - 8m_sm_c^2 + 8m_s^2m_c - 16m_s^3)I_2(1, 3, 2) + (8m_c^5 - 8m_sm_c^4 - 16m_s^2m_c^3 \\
& + 16m_s^3m_c^2 + 8m_s^4m_c - 8m_s^5)I_2(1, 3, 3) + (24m_c^2m_s - 24m_c^3)I_2(1, 4, 1) \\
& + (8m_b - 8m_s)I_2(2, 1, 2) + (16m_sq^2 - 40m_s^3)I_2(2, 1, 3) + (8m_b - 48m_c \\
& + 40m_s)I_2(3, 1, 1) + (8m_b^3 - 8m_sm_b^2 + 8m_s^2m_b - 8q^2m_b - 16m_s^3 + 16m_sq^2)I_2(3, 1, 2) \\
& + (-8m_s^5 + 16m_b^2m_s^3 + 16q^2m_s^3 - 8m_b^4m_s - 8q^4m_s + 16m_b^2q^2m_s)I_2(3, 1, 3) \\
& + (72m_b^2m_c - 72m_b^2m_s)I_2(4, 1, 1) \Big\} \\
& + D_0^1 \Big\{ -24m_c^3I(1, 4, 1) - 24m_s^2m_cI(1, 1, 4) + 72m_b^2I(4, 1, 1)m_c + (24m_b - 16m_c \\
& + 24m_s)I(1, 1, 3) + (16m_b - 16m_c + 24m_s)I(1, 2, 2) + (-32m_c^3 + 48m_bm_c^2 + 56m_sm_c^2 \\
& - 32m_s^2m_c + 16q^2m_c - 32m_bm_sm_c + 24m_s^3 + 40m_bm_s^2 + 8m_sq^2)I(1, 2, 3) \\
& + (24m_b - 32m_c)I(1, 3, 1) + (-16m_c^3 + 40m_bm_c^2 + 32m_sm_c^2 - 32m_s^2m_c + 16q^2m_c \\
& - 32m_bm_sm_c + 48m_bm_s^2)I(1, 3, 2) + (-16m_c^5 + 24m_bm_c^4 + 32m_sm_c^4 - 32m_s^2m_c^3 \\
& + 16q^2m_c^3 - 32m_bm_sm_c^3 + 32m_s^3m_c^2 + 16m_bm_s^2m_c^2 - 16m_s^4m_c - 32m_bm_s^3m_c \\
& + 16m_s^2q^2m_c + 24m_bm_s^4)I(1, 3, 3) + (8m_b + 8m_c)I(2, 2, 1) \\
& + (24m_c^3 + 16m_b^2m_c)I(2, 3, 1) + (8m_b - 40m_c)I(3, 1, 1) + (8m_b^3 + 8m_cm_b^2 + 8m_c^2m_b \\
& + 16m_c^3)I(3, 2, 1) + (8m_c^5 - 16m_b^2m_c^3 + 8m_b^4m_c)I(3, 3, 1) + (8m_s - 8m_b)I_1(1, 1, 3)
\end{aligned}$$

$$\begin{aligned}
& + (24m_b m_s^2 - 24m_c m_s^2)I_1(1, 1, 4) + (-8m_b - 24m_c)I_1(1, 3, 1) + (24m_b m_c^2 \\
& - 24m_c^3)I_1(1, 4, 1) + (8m_s - 8m_b)I_1(2, 1, 2) + (24m_s^3 - 24m_b m_s^2 - 8m_b^2 m_s - 8q^2 m_s \\
& + 16m_b q^2)I_1(2, 1, 3) + (-16m_b^3 + 8m_c m_b^2 - 8m_c^2 m_b + 16m_c^3)I_1(2, 3, 1) \\
& + (16m_b - 40m_c)I_1(3, 1, 1) + (-24m_b^3 - 16m_s^2 m_b + 16q^2 m_b)I_1(3, 1, 2) \\
& + (-8m_b^5 + 16m_s^2 m_b^3 + 16q^2 m_b^3 - 8m_s^4 m_b - 8q^4 m_b + 16m_s^2 q^2 m_b)I_1(3, 1, 3) \\
& + (-16m_b^3 + 8m_c m_b^2 - 8m_c^2 m_b + 16m_c^3)I_1(3, 2, 1) + (-8m_b^5 + 8m_c m_b^4 + 16m_c^2 m_b^3 \\
& - 16m_c^3 m_b^2 - 8m_c^4 m_b + 8m_c^5)I_1(3, 3, 1) + (72m_b^2 m_c - 72m_b^3)I_1(4, 1, 1) + (-16m_b \\
& + 8m_c + 32m_s)I_2(1, 1, 3) + (24m_s^3 + 48m_b m_s^2 - 72m_c m_s^2)I_2(1, 1, 4) \\
& + (8m_s - 8m_c)I_2(1, 2, 2) + (16m_c^3 - 8m_s m_c^2 + 8m_s^2 m_c + 16q^2 m_c - 16m_s^3 \\
& - 8m_s q^2)I_2(1, 2, 3) + (-16m_b - 72m_c - 8m_s)I_2(1, 3, 1) + (16m_c^3 - 8m_s m_c^2 + 8m_s^2 m_c \\
& + 8q^2 m_c - 16m_s^3 - 16m_s q^2)I_2(1, 3, 2) + (8m_c^5 - 8m_s m_c^4 - 16m_s^2 m_c^3 + 16q^2 m_c^3 \\
& + 16m_s^3 m_c^2 - 16m_s q^2 m_c^2 + 8m_s^4 m_c + 16m_s^2 q^2 m_c - 8m_s^5 - 16m_s^3 q^2)I_2(1, 3, 3) \\
& + (-72m_c^3 + 48m_b m_c^2 + 24m_s m_c^2)I_2(1, 4, 1) + (8m_s - 8m_b)I_2(2, 1, 2) + (8m_s^3 \\
& - 48m_b m_s^2 - 16m_b^2 m_s + 32m_b q^2)I_2(2, 1, 3) + (-32m_b^3 + 16m_c m_b^2 - 16m_c^2 m_b \\
& + 32m_c^3)I_2(2, 3, 1) + (40m_b - 128m_c + 40m_s)I_2(3, 1, 1) + (-40m_b^3 - 8m_s m_b^2 \\
& - 24m_s^2 m_b + 24q^2 m_b - 16m_s^3 + 16m_s q^2)I_2(3, 1, 2) + (-16m_b^5 - 8m_s m_b^4 + 32m_s^2 m_b^3 \\
& + 32q^2 m_b^3 + 16m_s^3 m_b^2 + 16m_s q^2 m_b^2 - 16m_s^4 m_b - 16q^4 m_b + 32m_s^2 q^2 m_b - 8m_s^5 \\
& - 8m_s q^4 + 16m_s^3 q^2)I_2(3, 1, 3) + (-32m_b^3 + 16m_c m_b^2 - 16m_c^2 m_b + 32m_c^3)I_2(3, 2, 1) \\
& + (-16m_b^5 + 16m_c m_b^4 + 32m_c^2 m_b^3 - 32m_c^3 m_b^2 - 16m_c^4 m_b + 16m_c^5)I_2(3, 3, 1) \\
& + (-144m_b^3 + 216m_c m_b^2 - 72m_s m_b^2)I_2(4, 1, 1) \Big\} \tag{34}
\end{aligned}$$

where

$$D_i^j [I_n(M_1^2, M_2^2)] = (M_1^2)^i (M_2^2)^j \frac{\partial_i}{\partial(M_1^2)^i} \frac{\partial^j}{\partial(M_2^2)^j} [(M_1^2)^i (M_2^2)^j I_n(M_1^2, M_2^2)] .$$